THE PLOT
Pragmatics as the “science of the unsaid”.
Conversational Implicatures

A form of inference that arises from expectations about how speakers will behave, based on general assumptions about cooperative behavior.

Grice (1975)
Conversational Implicatures

A form of inference that arises from expectations about how speakers will behave, based on general assumptions about cooperative behavior.

(1) A. I am out of gas.
    B. There is a gas station around the corner.

\[ \sim \textit{the gas station is open and serviceable} \]

Grice (1975)
Informativity Implicatures

(2) **Strong Implicatures**

A. How many children do you have?
B. I have three children.

$\sim B$ doesn’t have four children

a.k.a. Scalar Implicatures
Informativity Implicatures

(2) **Strong Implicatures**
   A. How many children do you have?
   B. I have three children.
   \[\sim B \text{ doesn’t have four children}\]

(3) **Weak Implicatures**
   A. Where is Sue? I need to track her down.
   B. She is in Europe.
   \[\sim B \text{ doesn’t know where exactly Sue is}\]
(4) a. Sue ate broccoli or banana.

\( \sim \) *Broccoli? Banana? Both?*
Dedicated expressions of ignorance

(4) a. Sue ate broccoli or banana.
   ⟷ Broccoli? Banana? Both?

b. I saw some kind of contraption in the copy-room.
   ⟷ Copy machine? High-res scanner? Paper shredder?
Dedicated expressions of ignorance

(4) a. Sue ate broccoli or banana.
   \( \rightsquigarrow \) Broccoli? Banana? Both?

b. I saw some kind of contraption in the copy-room.
   \( \rightsquigarrow \) Copy machine? High-res scanner? Paper shredder?

c. Sue paid twenty-some dollars for the book.
   \( \rightsquigarrow \) 21? 22? 23? ... 29?
Dedicated expressions of ignorance

(4) a. Sue ate broccoli or banana.
   \[ \sim Broccoli? Banana? Both? \]

b. I saw some kind of contraption in the copy-room.
   \[ \sim Copy machine? High-res scanner? Paper shredder? \]

c. Sue paid twenty-some dollars for the book.
   \[ \sim 21? 22? 23? \ldots 29? \]

d. Al arrived at least to the base camp
   \[ \sim Camp 1? Camp 2? Camp 3? Summit? \]
Dedicated expressions of ignorance

(4) a. Sue ate broccoli or banana.
   \( \sim \) Broccoli? Banana? Both?

b. I saw some kind of contraption in the copy-room.
   \( \sim \) Copy machine? High-res scanner? Paper shredder?

c. Sue paid twenty-some dollars for the book.
   \( \sim \) 21? 22? 23? ... 29?

d. Al arrived at least to the base camp
   \( \sim \) Camp 1? Camp 2? Camp 3? Summit?

\( \Rightarrow \) These are all \textbf{Ignorance Inferences} (II\textsubscript{s}).
General Questions

**Empirical**
What propositions **can** and **must** we be ignorant about to felicitously use “ignorance inducing” expressions?

**Theoretical**
What is the right (formal) characterization of IIs? How do we explain convergence among speakers?
Today: the scalar modifier *at least* (AL)

- Why AL?
  - Flexibility: modifies *all* sorts of constituents.
  - Robust: IIs of AL are “obligatory”.
Today: the scalar modifier *at least* (AL)

- **Why AL?**
  - Flexibility: modifies *all* sorts of constituents.
  - Robust: IIs of AL are “obligatory”.

- **First: DESCRIPTION**
  - Introduce the basic pragmatic properties of the IIs induced by AL.
  - Closing the descriptive gap between AL modifying numerals and other cases.
Today: the scalar modifier *at least* (AL)

- **Why AL?**
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  - Robust: IIs of AL are “obligatory”.

- **First: DESCRIPTION**
  - Introduce the basic pragmatic properties of the IIs induced by AL.
  - Closing the descriptive gap between AL modifying numerals and other cases.

- **Second: THEORY**
  - A Gricean pragmatic calculus that accounts for all the data.
  - Two sources of alternatives; emphasis on the role of *order* over *entailment*.
Not today!

Not all constructions involving AL convey ignorance.

1. Concessive AL

(5) Sue’s flight got canceled but at least she was reimbursed.
Not all constructions involving AL convey ignorance.

① Concessive AL

(5) Sue’s flight got canceled but at least she was reimbursed.

② Embedded AL

(6) a. Every student wrote at least two papers.
   b. You must write at least two QPs to be ABD.
THE BASICS OF AT LEAST
At least (AL)

Three undisputed facts about AL and AL-statements:

① AL may combine with a variety of expressions.
② AL denotes a lower bound.
③ AL-statements come with IIs.
At least (AL)

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(7) Sue has at least four dogs
⇒ The speaker knows that Sue does not have less than four dogs
∼ The speaker is ignorant about the exact number of dogs
At least (AL)

Three undisputed facts about AL and AL-statements:

1. AL may combine with a variety of expressions.
2. AL denotes a lower bound.
3. AL-statements come with II’s.

(7) Sue has at least four dogs
    ⇒ The speaker knows that Sue does not have less than four dogs
    ∼> The speaker is ignorant about the exact number of dogs

(8) At least Liz and Sue read the paper
    ⇒ The speaker knows that both Liz and Sue read the paper
    ∼> The speaker is ignorant about exactly who read the paper
Agreements and Disagreements

Somewhat disputed is what the exact division of labor between semantics and pragmatics is. A commonly held view:

- The lower bound is part of the *semantic* content of AL.
- IIs are *pragmatic*. 
Agreements and Disagreements

Somewhat disputed is what the exact division of labor between semantics and pragmatics is. A commonly held view:

- The lower bound is part of the *semantic* content of AL.
- IIIs are *pragmatic*.

Much disputed is *how* to derive the inferences:

- AL as disjunction [Büring 2007]
- AL as a (double) modal [Geurts and Nouwen 2007]
- AL as a meta-speech act operator [Cohen and Krifka 2014]
- AL as an epistemic indefinite [Nouwen 2015]
- AL as “inquisitive” [Coppock and Brochhagen 2013b,a]
- AL as (more or less) “classical” quantifier:
  - (Neo-)Gricean [Mendia 2015, Kennedy 2015, Schwarz 2016]
  - Grammatical [Mayr and Meyer 2014, Buccola and Haida 2019]
The descriptive gap

Most attention has been devoted to AL modifying numerals. AL may combine with a number of different types of complements or associates, consistently leading to IIs in all these environments.

(9) a. At least some students came to the party. [HORN SCALES]
    \(\sim\) the speaker S is ignorant about whether all students came
b. At least Bill and Sue came to the party. [CARDINALITY SCALES]
    \(\sim\) S is ignorant about whether someone else came to the party
c. Sue won at least the silver medal. [LEXICAL SCALES]
    \(\sim\) S is ignorant about whether Sue won the gold medal
d. Bill ate at least broccoli. [EVALUATIVE SCALES]
    \(\sim\) S is ignorant about whether Bill ate candy

Two questions

Predictability

Is there any proposition in particular about which the speaker must be ignorant so that they can successfully use an AL-statement? If so, what are they?
Two questions

Predictability

Is there any proposition in particular about which the speaker must be ignorant so that they can successfully use an AL-statement? If so, what are they?

Uniformity

Are the inferences that come with AL the same across the board, regardless of its type of associate?
CHARACTERIZING IGNORANCE
Total vs Partial Ignorance

Disjunction demands “equal rights for each disjunct”: A speaker uttering (10) is committed to be ignorant about each individual disjunct.

(10) Liz read Tintin, Asterix, or Gaston.

\[ S \text{ doesn’t know whether Liz read Tintin} \]
\[ S \text{ doesn’t know whether Liz read Asterix} \]
\[ S \text{ doesn’t know whether Liz read Gaston} \]

Total vs Partial Ignorance

Disjunction demands “equal rights for each disjunct”: A speaker uttering (10) is committed to be ignorant about each individual disjunct.

(10) Liz read Tintin, Asterix, or Gaston.

\(~ S \text{ doesn’t know whether Liz read Tintin}\)
\(~ S \text{ doesn’t know whether Liz read Asterix}\)
\(~ S \text{ doesn’t know whether Liz read Gaston}\)

This is **Total Ignorance.**

Total vs Partial Ignorance

Indefinites do not impose such stringent restrictions:

(11) Liz paid twenty-some dollars for the book, but less than 27.
\[ \sim 21? \ 22? \ 23? \ ... \ 26? \]

(12) Some friend of yours called; it wasn’t Louise.
\[ \sim Cynthia? \ Liz? \ Zoe? \]
Indefinites do not impose such stringent restrictions:

(11) Liz paid twenty-some dollars for the book, but less than 27.
    \(\sim 21?\ 22?\ 23?\ \ldots\ 26?\)

(12) Some friend of yours called; it wasn’t Louise.
    \(\sim Cynthia?\ Liz?\ Zoe?\)

The speaker is unable to identify the witness of an existential claim, but ignorance is not required about every individual in the domain.

\(\checkmark\) This is **Partial Ignorance**.
Total vs Partial Ignorance

(13) Last night LeBron James scored at least 30 points in three-pointers.

Total vs Partial Ignorance

(13) Last night LeBron James scored at least 30 points in three-pointers.

(14) [We forgot the password of the WIFI network. All we know is that passwords are between 4 and 10 characters long.]
The password is at least six characters long.

Total vs Partial Ignorance

(13) Last night LeBron James scored at least 30 points in three-pointers.

(14) [We forgot the password of the WIFI network. All we know is that passwords are between 4 and 10 characters long.] 
The password is at least six characters long.

Partial Ignorance

Assertibility conditions of AL

For a statement of the form ‘at least \( P \)’, we want to know:

① Is it compatible with knowing ‘more than \( P \)’?
Assertibility conditions of AL

For a statement of the form "at least $P$", we want to know:

1. Is it compatible with knowing "more than $P$"?
2. Is it compatible with knowing "only $P$"?
Assertibility conditions of AL

For a statement of the form ‘at least $P$’, we want to know:

1. Is it compatible with knowing ‘more than $P$’?
2. Is it compatible with knowing ‘only $P$’?
3. Is it compatible with knowing any other alternative?
Assertibility conditions of AL

For a statement of the form «at least P», we want to know:

1. Is it compatible with knowing «more than P»?
2. Is it compatible with knowing «only P»?
3. Is it compatible with knowing any other alternative?

(15) «at least P»...
For a statement of the form 「at least $P$」, we want to know:

1. Is it compatible with knowing 「more than $P$」?
2. Is it compatible with knowing 「only $P$」?
3. Is it compatible with knowing any other alternative?

(15) 「at least $P$」…
   a. 「and ______」 $\vdash$ 「more than $P$」

①
Assertibility conditions of AL

For a statement of the form \( \text{at least } P \), we want to know:

1. Is it compatible with knowing \( \text{more than } P \)?
2. Is it compatible with knowing \( \text{only } P \)?
3. Is it compatible with knowing any other alternative?

(15) \( \text{at least } P \)...
   a. \( \text{and } \_\_\_\_\_\_\_\text{ and } \_\_\_\_\_\_\_\text{ } \models \text{more than } P \) \( \uparrow \) \( \text{①} \)
   b. \( \text{and } \_\_\_\_\_\_\_\text{ and } \_\_\_\_\_\_\_\text{ } \models \text{only } P \) \( \uparrow \) \( \text{②} \)
Assertibility conditions of AL

For a statement of the form ‘at least P’, we want to know:

① Is it compatible with knowing ‘more than P’?
② Is it compatible with knowing ‘only P’?
③ Is it compatible with knowing any other alternative?

(15) ‘at least P’...
   a. ‘ and ______’ ⊧ ‘more than P’ ①
   b. ‘ and ______’ ⊧ ‘only P’ ②
   c. ‘ and ______’ ⊭ ‘more than P’, ‘only P’ ③
Assertibility conditions of AL

(16) Sue ate at least two apples...
Assertibility conditions of AL

(16) Sue ate at least two apples...
   a. # but she didn’t eat only two. ⇒ S knows ‘more than 2’
### Assertibility conditions of AL

(16) Sue ate at least two apples...

a. \# but she didn’t eat only two.  
   \[ \Rightarrow S \text{ knows } 'more than 2' \]
   \[ \times \]

b. \# and she did not eat more than two.  
   \[ \Rightarrow S \text{ knows } 'exactly 2' \]
   \[ \times \]
(16) Sue ate at least two apples...
   a. # but she didn’t eat only two.  
      ⇒ S knows ‘more than 2’  
      ✓
   b. # and she did not eat more than two.  
      ⇒ S knows ‘exacty 2’  
      ✓
   c. but she didn’t eat {four/three or four/six/...}.  
      ✓
Thus, for an AL-statement *at least* $P$, the speaker *must be* ignorant as to whether:

- *only* $P$ is the case.
- *more than* $P$ is the case.

---

Beyond numerals

Some non-numeral cases pattern with numerals:

(17) Sue ate at least some of the apples...
Beyond numerals

Some non-numeral cases pattern with numerals:

(17) Sue ate at least some of the apples...
   a. ≠ but I know that she didn’t eat just some.
      ⇒ S knows that ‘more than just some’

Beyond numerals

Some non-numeral cases pattern with numerals:

(17) Sue ate at least some of the apples...

a.  but I know that she didn’t eat just some.
    ⇒ S knows that ‘more than just some’ ×

b.  but she didn’t eat all.
    ⇒ S knows ‘just some’ ×
Beyond numerals

Numerals and quantifiers like *some* share two important logical properties:

1. Asymmetric logical entailment.
2. Total order.
Beyond numerals

Numerals and quantifiers like *some* share two important logical properties:

1. Asymmetric logical entailment.
2. Total order.

Total Orders

\[
\begin{array}{ccc}
4 & \rightarrow & \text{all} \\
3 & \rightarrow & \text{most} \\
2 & \rightarrow & \text{many} \\
1 & \rightarrow & \text{some}
\end{array}
\]
Beyond numerals

Numerals and quantifiers like *some* share two important logical properties:

1. Asymmetric logical entailment.
2. Total order.

Total Orders

<table>
<thead>
<tr>
<th>4</th>
<th>all</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>most</td>
</tr>
<tr>
<td>2</td>
<td>many</td>
</tr>
<tr>
<td>1</td>
<td>some</td>
</tr>
</tbody>
</table>

Which property is responsible for the exact form of AL’s IIIs?

Which of the two notions *matters* for implicature calculation?
Contextual entailment

Contextual entailment = Common Ground information + Logical entailment

(18) Sue has a college degree \(\neq\) Sue has a high-school degree.
Contextual entailment

Contextual entailment = Common Ground information + Logical entailment

(18) Sue has a college degree \( \neq \) Sue has a high-school degree.

(19) \[
\text{Sue has a college degree} \quad \frac{\text{Sue has a college degree}}{\text{Sue has a high school degree}}
\]
Contextual entailment

Contextual entailment = Common Ground information + Logical entailment

(18) Sue has a college degree $\not\equiv$ Sue has a high-school degree.

(19) \[
\begin{align*}
&\text{Sue has a college degree} \\
\therefore &\text{Sue has a high school degree}
\end{align*}
\]

(20) \[
\begin{align*}
&\text{All college graduates have high school degrees} \\
\therefore &\text{Sue has a high school degree}
\end{align*}
\]
Contextual entailment

(21) Sue has at least a high school degree...
(21) Sue has at least a high school degree...
   a. ≠ and she has a college degree.
      ⇒ S knows that "more than just HS"
(21) Sue has at least a high school degree...
   a. # and she has a college degree.
      ⇒ S knows that ‘more than just HS’  
      ✗
   b. # but she didn’t go to college.
      ⇒ S knows that ‘only HS’  
      ✗
Contextual entailment

(21) Sue has at least a high school degree...
   a. # and she has a college degree.  
      ⇒ S knows that $more \ than \ just \ HS$  
      √
   b. # but she didn’t go to college.  
      ⇒ S knows that $only \ HS$  
      ×
   c. but I know that she does not have a PhD.  
      √
Contextual entailment

(21) Sue has at least a high school degree...
   a.  # and she has a college degree.
        ⇒ S knows that "more than just HS"

   b.  # but she didn’t go to college.
        ⇒ S knows that "only HS"

   c.  but I know that she does not have a PhD.

Like with numerals/quantifiers, the speaker is ignorant as to whether:

- only P is the case.
- more than P is the case.
No entailment

Logically and contextually independent alternatives can also be ordered:

Medals & Professorships

- gold
- full
- ≤ silver associate
- bronze assistant
No entailment

(22) Sue won the gold medal ≠ Sue won the silver medal
No entailment

(22) Sue won the gold medal \( ∉ \) Sue won the silver medal

(23) 
\[
\begin{align*}
\text{Sue won the gold medal} & \\
\therefore \text{Sue won the silver medal}
\end{align*}
\]

\[\text{[CG???]}\]

No amount of Common Ground information may be supplied so as to make *gold* entail *silver*, these are *logically* and *contextually* independent.
No entailment

(24) Sue won at least the bronze medal...
(24) Sue won at least the bronze medal...
   a. she placed first or second.
      ⇒ S knows that \textit{more than bronze}
No entailment

(24) Sue won at least the bronze medal...
   a. # she placed first or second. 
      ⇒ S knows that 'more than bronze'
   b. # she didn’t place first or second. 
      ⇒ S knows that 'only bronze'
No entailment

(24) Sue won at least the bronze medal...

a. ¬ she placed first or second.
   ⇒ S knows that "more than bronze"
   \[\times\]

b. ¬ she didn’t place first or second.
   ⇒ S knows that "only bronze"
   \[\times\]

c. but I know that she did not place second.
   \[\checkmark\]
No entailment

(24) Sue won at least the bronze medal...
   a.  she placed first or second.
       ⇒ S knows that \( more \text{ than bronze} \)
       \( \times \)
   b.  she didn’t place first or second.
       ⇒ S knows that \( only \text{ bronze} \)
       \( \times \)
   c.  but I know that she did not place second.
       \( \checkmark \)

- Once more, the speaker is ignorant as to whether:
  - \( only \ P \) is the case.
  - \( more \text{ than} \ P \) is the case.
No entailment

(24) Sue won at least the bronze medal...
   a.  ≠ she placed first or second.
       ⇒ S knows that 'more than bronze' ×
   b.  ≠ she didn’t place first or second.
       ⇒ S knows that 'only bronze' ×
   c.  but I know that she did not place second. ✓

● Once more, the speaker is ignorant as to whether:
   ▶ only P is the case.
   ▶ more than P is the case.

😊 Conclusion about entailment

AL is blind to entailment (contextual or logical), all it needs is an ordered structure.
Ordering: Partial Orders

If AL is blind about **entailment**, is it also blind about different types of order?
If AL is blind about **entailment**, is it also blind about different types of order?

A Partial Order

- Bill ⊕ Sue ⊕ Mary
- Bill ⊕ Sue ⊕ Mary
- Bill ⊕ Mary ⊕ Mary
- Bill ⊕ Sue ⊕ Mary
- Bill ⊕ Sue ⊕ Mary
- Bill ⊕ Sue ⊕ Mary
- Bill ⊕ Sue ⊕ Mary
- Bill ⊕ Sue ⊕ Mary
- Bill ⊕ Sue ⊕ Mary
Ordering: Partial Orders

**Context:** Sherlock Holmes went on vacation for a couple of days and let some of his friends celebrate dinner at 221B Baker Street: Dr. Watson, Mrs. Hudson, Mycroft, Irene Adler and some of the Baker Street Irregulars. After vacation, he returns to his room only to discover that somebody has been messing with his chemistry set. Inspector Lestrade from Scotland Yard is with him, and asks: *Who do you think touched the chemistry set?*

(25) It was at least Mycroft and Mrs. Hudson...
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(25) It was at least Mycroft and Mrs. Hudson...
   a. but certainly not only them.
   ⇒ S knows that \(\text{more than } M\&H\)
**Context:** Sherlock Holmes went on vacation for a couple of days and let some of his friends celebrate dinner at 221B Baker Street: Dr. Watson, Mrs. Hudson, Mycroft, Irene Adler and some of the Baker Street Irregulars. After vacation, he returns to his room only to discover that somebody has been messing with his chemistry set. Inspector Lestrade from Scotland Yard is with him, and asks: *Who do you think touched the chemistry set?*

(25) It was at least Mycroft and Mrs. Hudson...

a. but certainly not only them.  
   \[ \Rightarrow S \text{ knows that } \text{``more than M&H''} \]

b. # and nobody else.  
   \[ \Rightarrow S \text{ knows that } \text{``only M&H''} \]

\[ \checkmark \text{  } \times \]
**Ordering: Partial Orders**

**Context:** Sherlock Holmes went on vacation for a couple of days and let some of his friends celebrate dinner at 221B Baker Street: Dr. Watson, Mrs. Hudson, Mycroft, Irene Adler and some of the Baker Street Irregulars. After vacation, he returns to his room only to discover that somebody has been messing with his chemistry set. Inspector Lestrade from Scotland Yard is with him, and asks: *Who do you think touched the chemistry set?*

(25) It was at least Mycroft and Mrs. Hudson...

a. but certainly not only them. 
   ⇒ $S$ knows that $'more \ than \ M&H'$ ✔

b. # and nobody else.
   ⇒ $S$ knows that $'only \ M&H'$ ✗

c. but I know that it wasn’t Irene Adler. ✔
(26) a. Who touched the chemistry set?
   b. At least Mycroft and Mrs. Hudson, but certainly not only them.

   SH knows that ‘only M&H’ is false  ✓
Ordering: Total vs. Partial

(26) a. Who touched the chemistry set?
   b. At least Mycroft and Mrs. Hudson, but certainly not only them.

   \[SH \text{ knows that } \text{"} \text{only M&H} \text{"} \text{ is false}\]

   ✓

(27) a. How many people touched the chemistry set?
   b. At least two people, but certainly not just two.

   \[SH \text{ knows that } \text{"} \text{only 2} \text{"} \text{ is false}\]

   ✗
(28) *Assertibility conditions of ‘at least P’ with Total Orders:*
(28) **Assertibility conditions of** 「at least P」**with Total Orders:**

a. S **must** be ignorant about 「more than P」.
(28) Assertibility conditions of ‘at least P’ with Total Orders:
   a. S **must** be ignorant about ‘more than P’.
   b. S **must** be ignorant about ‘only P’.

The use of AL when modifying Totally Ordered domains is more restricted wrt. the exhaustive interpretation of the prejacent.
Assertibility conditions

(28) Assertibility conditions of "at least P" with **Total Orders**:
   a. **must** be ignorant about "more than P".
   b. **must** be ignorant about "only P".

(29) Assertibility conditions of "at least P" with **Partial Orders**:
   a. **must** be ignorant about "more than P".
Assertibility conditions

(28) Assertibility conditions of "at least $P$" with **Total Orders**:
   a. $S$ **must** be ignorant about "more than $P$".
   b. $S$ **must** be ignorant about "only $P$".

(29) Assertibility conditions of "at least $P$" with **Partial Orders**:
   a. $S$ **must** be ignorant about "more than $P$".
   b. $S$ **may but need not** be ignorant about "only $P$".
Assertibility conditions

(28) *Assertibility conditions of* ‘at least P’ *with Total Orders:*
   a. S **must** be ignorant about ‘more than P’.
   b. S **must** be ignorant about ‘only P’.

(29) *Assertibility conditions of* ‘at least P’ *with Partial Orders:*
   a. S **must** be ignorant about ‘more than P’.
   b. S **may but need not** be ignorant about ‘only P’.

☞ The use of AL when modifying Totally Ordered domains is more restricted wrt. the **exhaustive interpretation of the prejacent.**
Interim Conclusion

① Alternatives of AL-statements need not be ordered by (logical/contextual) entailment.

② Alternatives must be ordered by some (possibly ad hoc) relation ≤.

③ Structural differences between orderings of alternatives correlate with different IIs.
Predictability

Is there any proposition in particular about which the speaker must be ignorant so that they can successfully use an AL-statement?

Yes.

IIs indicate the minimal epistemic conditions that speakers must meet to successfully use AL; in turn, these minimal conditions constitute what is minimally predictable about the speaker’s epistemic state.
Answer to Uniformity

**Uniformity**

Are the inferences that come with AL the same across the board, regardless of its type of associate?

- No.
### New Generalization

The exact form of AL’s II’s depends on the **order** of AL’s associate.
CALCULATING INFERENCES
An observation

AL is Conventionally Associated with Focus: IIs vary accordingly.

(30) a. The chair at least invited the \text{postdoc}_F to lunch.
    \[ \sim \text{ignorance about whether someone else was invited} \]

b. The chair at least invited the postdoc \text{to lunch}_F.
    \[ \sim \text{ignorance about whether she got invited to something else} \]

---

Beaver and Clark (2008), Coppock and Brochhagen (2013b), Mendia (2017)
Semantics: focus

A two tier semantic system delivering an ordinary semantic value $\llbracket \cdot \rrbracket^o$ and a focus semantic value $\llbracket \cdot \rrbracket^f$.

(31) Sue ate broccoli$_F$

a. $\llbracket [Sue \text{ ate } \text{BROCCOLI}]_F \rrbracket^o = \text{ate}(Sue, \text{broccoli})$

b. $\llbracket [Sue \text{ ate } \text{BROCCOLI}]_F \rrbracket^f = \{ x \in D_e \mid \text{ate}(Sue, x) \}$

Rooth (1985, 1992) et seq.
Semantics: focus

A two tier semantic system delivering an ordinary semantic value $\llbracket \cdot \rrbracket^o$ and a focus semantic value $\llbracket \cdot \rrbracket^f$.

(31) Sue ate \underline{broccoli}$^F$

a. $\llbracket \llbracket Sue \text{ ate } \llbracket \text{BROCCOLI}\rrbracket^F \rrbracket^o = \text{ate}(Sue, \text{broccoli})$

b. $\llbracket \llbracket Sue \text{ ate } \llbracket \text{BROCCOLI}\rrbracket^F \rrbracket^f = \{x \in D_e \mid \text{ate}(Sue, x)\}$

A contextually determined set of relevant alternatives $C$:

(32) A proposition with some focused constituent $\varphi^F$ is defined only if $C$ is a subset of $\llbracket \varphi \rrbracket^f$ containing $\llbracket \varphi \rrbracket^o$ and at least one other element.

Rooth (1985, 1992) et seq.
Semantics: AL

(33) \([S_1 \text{ at least } C [S_2 [S_3 \text{ Sue is [an assistant professor]}_F ] \sim C]]\)

(34) \([\text{at least}] = \lambda C_{\langle \text{st}, t \rangle} . \lambda p_{\langle \text{st} \rangle} . \lambda w . \exists q [q \in C \land q(w) \land p \leq q]\)

(35) a. \([33]^f = \{ P(\text{Sue}) \mid P \in D_{\langle \text{e}, t \rangle}\}\)
b. \([33]^o = \text{Sue is an assistant professor}\)
c. \([33]^{cf} = \{\ldots \text{Sue is a visiting professor,} \}
\text{Sue is an assistant professor,}
\text{Sue is an associate professor,} \ldots \}

\([33] = \lambda w . \exists q [q \in [33]^{cf} \land q(w) \land [(33)]^o \leq q]\)
Semantics: AL

(33) \([S_1 \text{ at least } C [S_2 [S_3 \text{ Sue is [an assistant professor]}_F ] \sim C]]\)

(34) \([\text{at least}] = \lambda C_{(st,t)} \cdot \lambda p_{(st)} \cdot \lambda w . \exists q [q \in C \land q(w) \land p \leq q]\)

(35) a. \([33]^f = \{P(\text{Sue}) \mid P \in D_{(e,t)}\}\)
    b. \([33]^o = \text{Sue is an assistant professor}\)
    c. \([33]_{cf} = \begin{cases} ...
                     & \text{Sue is a visiting professor,} \\
                     & \text{Sue is an assistant professor,} \\
                     & \text{Sue is an associate professor,} \\
                     ...
                   \end{cases}\)

\([33] = \lambda w . \exists q [q \in [33]_{cf} \land q(w) \land [(33)]^o \leq q]\)
Pragmatics: Preliminaries

- I enclose propositions in square brackets, such that $[\varphi]$ stands for some proposition containing an expression $\varphi$. For instance:
  - $[4]$  
    - 4 students came.
  - $[\text{Al} \oplus \text{Mary}]$, sometimes $[\text{A} \oplus \text{M}]$  
    - Al and Mary came.
Pragmatics: Preliminaries

• I enclose propositions in square brackets, such that \([\varphi]\) stands for some proposition containing an expression \(\varphi\). For instance:
  - \([4]\) \(4 \text{ students came.}\)
  - \([\text{Al} \oplus \text{Mary}], \text{sometimes } [A \oplus M]\) \(\text{Al and Mary came.}\)

• With modifiers, I use:
  - \([\geq \varphi]\) \([\text{at least } \varphi]\)
  - \([\text{EX } \varphi]\) \([\text{only/exactly } \varphi; \text{the exhaustive interpretation}]\)
I enclose propositions in square brackets, such that \([\varphi]\) stands for some proposition containing an expression \(\varphi\). For instance:

- \([4]\)  
  \(4\) students came.

- \([Al \oplus Mary]\), sometimes \([A \oplus M]\)
  \(Al\ \text{and}\ Mary\ \text{came}\).

With modifiers, I use:

- \([\geq \varphi]\)  
  \([\text{at least } \varphi]\)

- \([EX \varphi]\)  
  \([\text{only/exactly } \varphi;\ \text{the exhaustive interpretation}]\)

I also use the familiar epistemic operators \(K_S\) and \(P_S\):

- \(K_S[\varphi]\)  
  \(S\ \text{knows that } \varphi\)

- \(P_S[\varphi]\)  
  \(\varphi\ \text{is possible according to } S\)
Pragmatics: Ignorance

*Ignorance* is a stronger notion than mere *uncertainty*:

- Uncertainty: \( \neg K_S[\varphi] \)
- Ignorance: \( \neg K_S[\varphi] \land \neg K_S[\neg \varphi] \) \( \equiv P_S[\varphi] \land P_S[\neg \varphi] \)
Pragmatics: A basic Gricean-style calculus

(36) Sue read Tintin or Asterix.  \[\sim S \text{ doesn't know which}\]
Pragmatics: A basic Gricean-style calculus

(36) Sue read Tintin or Asterix.

a. ASSERTION: \([T \lor A]\)

\(\sim S \text{ doesn’t know which}\)
(36) Sue read Tintin or Asterix. ~S doesn’t know which

a. ASSERTION: [T ∨ A]
b. EPISTEMIC IMPLICATION: K_S [T ∨ A]
(36) Sue read Tintin or Asterix. \(\sim S\) doesn’t know which

a. **ASSERTION**: \([T \lor A]\)

b. **EPISTEMIC IMPLICATION**: \(K_S[T \lor A]\)

Utterance of a sentence \(\varphi\) by a speaker \(S\) commits \(S\) to the knowledge of \(\varphi\).
(36) Sue read Tintin or Asterix.  

~S doesn’t know which

a. ASSERTION: $[T \lor A]$

b. EPISTEMIC IMPLICATION: $K_S[T \lor A]$

Utterance of a sentence $\varphi$ by a speaker $S$ commits $S$ to the knowledge of $\varphi$.

c. $SA([T \lor A]) = \{[T], [A]\}$
Pragmatics: A basic Gricean-style calculus

(36) Sue read Tintin or Asterix.    \[ \sim S \text{ doesn’t know which} \]

a. **Assertion:** \([T \lor A]\)

b. **Epistemic Implication:** \(K_S[T \lor A]\)

Utterance of a sentence \(\varphi\) by a speaker \(S\) commits \(S\) to the knowledge of \(\varphi\).

c. \(SA([T \lor A]) = \{[T], [A]\}\)

A proposition \(\psi\) constitutes a Stronger Alternative to \(\varphi\) if it conveys more information (i.e. both being true, \(\psi\) is compatible with less situations). If \(\psi\) is relevant and true, \(S\) should choose \(\psi\) over \(\varphi\).
(36) Sue read Tintin or Asterix. \[ \sim S \text{ doesn’t know which} \]

a. **Assertion**: \[ [T \lor A] \]

b. **Epistemic Implication**: \[ K_S [T \lor A] \]

Utterance of a sentence \( \varphi \) by a speaker \( S \) commits \( S \) to the knowledge of \( \varphi \).

c. **SA**\( ([T \lor A]) = \{[T], [A]\} \)

A proposition \( \psi \) constitutes a Stronger Alternative to \( \varphi \) if it conveys more information (i.e. both being true, \( \psi \) is compatible with less situations). If \( \psi \) is relevant and true, \( S \) should choose \( \psi \) over \( \varphi \).

d. **Primary Implicatures**: \( \neg K_S [T] \land \neg K_S [A] \)
Pragmatics: A basic Gricean-style calculus

(36) Sue read Tintin or Asterix.  \( \sim S \) doesn’t know which

a. **Assertion**: \([T \lor A]\)

b. **Epistemic Implication**: \(K_S[T \lor A]\)

Utterance of a sentence \(\varphi\) by a speaker \(S\) commits \(S\) to the knowledge of \(\varphi\).

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d. **Primary Implicatures**: \(\neg K_S[T] \land \neg K_S[A]\)

The inference that \(\neg K_S[\varphi]\), for some Stronger Alternative \(\varphi\) of \(\psi\).
Pragmatics: A basic Gricean-style calculus

(36) Sue read Tintin or Asterix.  \( \sim S \) doesn't know which

a. ASSERTION: \([T \lor A]\)

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Utterance of a sentence \(\varphi\) by a speaker \(S\) commits \(S\) to the knowledge of \(\varphi\).

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d. PRIMARY IMPLICATURES: \(\neg K_S[T] \land \neg K_S[A]\)

The inference that \(\neg K_S[\varphi]\), for some Stronger Alternative \(\varphi\) of \(\psi\).

e. IMPLICATURE BASE: \(K_S[T \lor A] \land \neg K_S[T] \land \neg K_S[A]\)
Pragmatics: A basic Gricean-style calculus

(36) Sue read Tintin or Asterix.  

a. **Assertion:** $[T \lor A]$

b. **Epistemic Implication:** $K_S[T \lor A]$

Utterance of a sentence $\varphi$ by a speaker $S$ commits $S$ to the knowledge of $\varphi$.

c. $SA([T \lor A]) = \{[T], [A]\}$

A proposition $\psi$ constitutes a Stronger Alternative to $\varphi$ if it conveys more information (i.e. both being true, $\psi$ is compatible with less situations). If $\psi$ is relevant and true, $S$ should choose $\psi$ over $\varphi$.

d. **Primary Implicatures:** $\neg K_S[T] \land \neg K_S[A]$

The inference that $\neg K_S[\varphi]$, for some Stronger Alternative $\varphi$ of $\psi$.

e. **Implicature Base:** $K_S[T \lor A] \land \neg K_S[T] \land \neg K_S[A]$

⚠️ Ignorance requires **both** $\neg K_S[\varphi]$ and $\neg K_S\neg[\varphi]$!  

$(\equiv P_S\neg[\varphi] \land P_S[\varphi])$
Pragmatics: A basic Gricean-style calculus

Symmetric alternatives

assertion

[T ∨ A] \implies [T] [A]

exhaust the space of possibilities
Pragmatics: A basic Gricean-style calculus

Symmetric alternatives

assertion

\[ [T \lor A] \]

⇒

exhaust the space of possibilities

\[ [T] [A] \]

● Suppose \( K_S[A] \): then \( K_S[A] \land \neg K_S[A] \vdash \bot \)
Pragmatics: A basic Gricean-style calculus

Symmetric alternatives

 assertion exhaustion the space of possibilities

\[
[T \lor A] \implies [T] [A]
\]

- Suppose \(K_S[A]\): then \(K_S[A] \land \neg K_S[A] \models \bot\)
- Suppose \(K_S[\neg A]\): then \(K_S[T]\), and thus \(K_S[T] \land \neg K_S[T] \models \bot\)
Pragmatics: A basic Gricean-style calculus

Symmetric alternatives

<table>
<thead>
<tr>
<th>assertion</th>
<th>exhaust the space of possibilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>[T ∨ A]</td>
<td>[T] [A]</td>
</tr>
</tbody>
</table>

- Therefore, neither $K_S[A]$ nor $K_S¬[A]$ can be true: $¬K_S[A] ∧ ¬K_S¬[A]$
Pragmatics: A basic Gricean-style calculus

Symmetric alternatives

<table>
<thead>
<tr>
<th>assertion</th>
<th>exhaust the space of possibilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>([T \lor A])</td>
<td>([T] [A])</td>
</tr>
</tbody>
</table>

- Suppose \(K_S[A]\): then \(K_S[A] \land \neg K_S[A] \models \bot\)
- Suppose \(K_S\neg[A]\): then \(K_S[T]\), and thus \(K_S[T] \land \neg K_S[T] \models \bot\)
- Therefore, neither \(K_S[A]\) nor \(K_S\neg[A]\) can be true: \(\neg K_S[A] \land \neg K_S\neg[A]\)

The missing inferences from the IMPLICATURE BASE are entailed:

\[
\underbrace{K_S[T \lor A]}_{EI} \land \underbrace{\neg K_S[T]}_{PI} \land \underbrace{\neg K_S[A]}_{PI} \land \underbrace{\neg K_S\neg[T]}_{Entailment} \land \underbrace{\neg K_S\neg[A]}_{Entailment}
\]

Ignorance through symmetry!
Pragmatics: Two steps for AL's alternatives

Alternatives pertinent to the pragmatic calculus are obtained from two independent sources:

- Focus alternatives.
- Substitution of AL-propositions by their corresponding EX-propositions.
Pragmatics: Step I – Use focus alternatives

For a proposition of the form \([AL \ p]\), replace the prejacent \(p\) by every proposition in \([p]^{cf}\).

\[
\text{(37)} \quad \text{Alt}_{FOC}(\llbracket 33 \rrbracket) = \{\llbracket \text{at least } p \rrbracket | p \in \llbracket 33 \rrbracket^{cf}\} = \\
\begin{cases}
\text{...} \\
\text{Sue is at least a visiting professor,} \\
\text{Sue is at least an assistant professor,} \\
\text{Sue is at least an associate professor,} \\
\text{...}
\end{cases}
\]
Pragmatics: Step II – Trade-in with EX

AL-statements stand in an asymmetric entailment relation with EX-statements: trade AL for EX.

\[
(38) \quad \text{Alt}_{\text{EX}}([33]) = \{ [\text{only } p] \mid p \in [33]^c \} =
\]

\[
\ldots
Sue \text{ is only a visiting professor,}
Sue \text{ is only an assistant professor,}
Sue \text{ is only an associate professor,}
\ldots
\]
Pragmatics: Step III – Informativity

We now have lots of alternatives to $p$: $\text{Alt}_\text{EX} \cup \text{Alt}_\text{FOC}$:

$$\text{Alt}(\llbracket 33 \rrbracket) = \begin{cases} 
\text{Sue is at least an assistant professor,} \\
\text{Sue is at least an associate professor,} \\
\text{Sue is at least a full professor,} \\
\text{Sue is at least a distinguished professor} \\
\text{Sue is only a visiting professor,} \\
\text{Sue is only an assistant professor,} \\
\text{Sue is only an associate professor,} \\
\text{Sue is only a full professor,} \\
\text{Sue is only a distinguished professor} 
\end{cases}$$
Pragmatics: Step III – Informativity

- **Informativity**
  A proposition $\varphi$ is more informative than an alternative $\psi$ iff, both are true and $\varphi$ is compatible with less worlds/situations than $\psi$. 
Pragmatics: Step III – Informativity

- **Informativity**
  A proposition $\varphi$ is more informative than an alternative $\psi$ iff, both are true and $\varphi$ is compatible with less worlds/situations than $\psi$.

- $\lbrack \text{Assoc} \rbrack$ ranks higher than $\lbrack \text{Assist} \rbrack$
- $\lbrack \text{Assoc} \rbrack \neq \lbrack \text{Assist} \rbrack$
- But $\lbrack \geq \text{Assoc} \rbrack$ is more informative than $\lbrack \geq \text{Assis} \rbrack$: it excludes $\lbrack \text{EXAssist} \rbrack$. 
The final set of Stronger Alternatives of $p$:

$$\text{SA}(\llbracket 33 \rrbracket) = \begin{cases} 
\text{Sue is at least an associate professor,} \\
\text{Sue is at least a full professor,} \\
\text{Sue is at least a distinguished professor} \\
\text{Sue is only an assistant professor,} \\
\text{Sue is only an associate professor,} \\
\text{Sue is only a full professor,} \\
\text{Sue is only a distinguished professor} 
\end{cases}$$

This set of stronger alternatives is sufficient to derive the right kind of IIIs simply by following our previous pragmatic calculus.
Pragmatics: Calculating Implicatures

(41) Sue is at least as assistant professor.
(41) Sue is at least as assistant professor.
   a. ASSERTION: \([\geq \text{Assis}]\)
Pragmatics: Calculating Implicatures

(41) Sue is at least as assistant professor.
  a. ASSERTION: $\geq \text{Assis}$
  b. EPISTEMIC IMPLICATION: $K_S[\geq \text{Assis}]$
The Plot

The Basics of at least

Characterizing ignorance

Calculating inferences

Conclusion & Ramifications

Pragmatics: Calculating Implicatures

(41) Sue is at least as assistant professor.

a. ASSERTION: $[\geq \text{Assis}]$

b. EPISTEMIC IMPLICATION: $K_S[\geq \text{Assis}]$

c. $SA([\geq \text{Assis}]) = (40)$
Pragmatics: Calculating Implicatures

(41) Sue is at least as assistant professor.
   a. ASSERTION: $[\geq \text{Assis}]$
   b. EPISTEMIC IMPLICATION: $K_S[\geq \text{Assis}]$
   c. $SA([\geq \text{Assis}]) = (40)$
   d. PRIMARY IMPLICATURES:
      $\neg K_S[\geq \text{Assoc}] \land \neg K_S[\geq \text{Assis}] \land \neg K_S[\geq \text{Full}] \land \neg K_S[\geq \text{Dist}]
      \land \neg K_S[\text{Ex Assis}] \land \neg K_S[\text{Ex Assoc}] \land \neg K_S[\text{Ex Full}] \land \neg K_S[\text{Ex Dist}]$
Pragmatics: Calculating Implicatures

(41) Sue is at least as assistant professor.
   a. ASSERTION: \([\geq \text{Assis}]\)
   b. EPISTEMIC IMPLICATION: \(K_S[\geq \text{Assis}]\)
   c. \(SA([\geq \text{Assis}]) = (40)\)
   d. PRIMARY IMPLICATURES:
      \[\neg K_S[\geq \text{Assoc}] \land \neg K_S[\geq \text{Assis}] \land \neg K_S[\geq \text{Full}] \land \neg K_S[\geq \text{Dist}]\]
      \[\neg K_S[\text{Ex Assis}] \land \neg K_S[\text{Ex Assoc}] \land \neg K_S[\text{Ex Full}] \land \neg K_S[\text{Ex Dist}]\]

Together with the Epistemic Implication, these implicatures constitute the Implicature Base.

(42) IMPlicature BASE:
    \[K_S[\geq \text{Assis}] \land \neg K_S[\geq \text{Assoc}] \land \neg K_S[\geq \text{Assist}] \land \neg K_S[\geq \text{Full}] \land \neg K_S[\geq \text{Dist}] \land \neg K_S[\text{Ex Assis}] \land \neg K_S[\text{Ex Assoc}] \land \neg K_S[\text{Ex Full}] \land \neg K_S[\text{Ex Dist}]\]
Pragmatics: Entailments

Two of the stronger alternatives in the Implicature Base are symmetric: \([Ex \ Assis]\) and \([\geq \ Assoc]\).

\[
\begin{align*}
\geq \ Assis & \quad \Rightarrow \quad \text{assertion} \\
& \quad \text{exhaust the space of possibilities} \\
\end{align*}
\]

\[
\begin{align*}
Ex \ Assis & \quad \text{SA} \\
\geq \ Assoc & \quad \text{SA}
\end{align*}
\]

The speaker **must** contend that:

- It’s possible that Sue is just an assistant professor
- It’s possible that Sue is not just an assistant professor
- It’s possible that Sue is more than just an assistant professor
- It’s possible that Sue is not more than just an assistant professor
Pragmatics: Conclusion I

When AL modifies a totally ordered associate:

① Two alternatives **must** constitute epistemic possibilities for the speaker:

- The exhaustive interpretation of the prejacent ([Ex Assis])
- The immediately stronger AL-alternative ([≥ Assoc])
When AL modifies a totally ordered associate:

1. Two alternatives **must** constitute epistemic possibilities for the speaker:
   - The exhaustive interpretation of the prejacent ([*Ex Assis*])
   - The immediately stronger AL-alternative ([*≥ Assoc*])

2. Other alternatives **may but need not** constitute epistemic possibilities.
When AL modifies a totally ordered associate:

1. Two alternatives **must** constitute epistemic possibilities for the speaker:
   - The exhaustive interpretation of the prejacent ([Ex Assis])
   - The immediately stronger AL-alternative ([≥ Assoc])

2. Other alternatives **may but need not** constitute epistemic possibilities.

Partial Ignorance
Assume a context with domain \{Bill, Sue, Ed\}.

(43) Liz saw at least \([Bill]_F\)
Assume a context with domain \{\textit{Bill, Sue, Ed}\}.

(43) Liz saw at least \[\text{Bill}\]_F

The derivation of the ordinary and focus semantic values proceeds as usual.

(44) a. \[43\]_f = \{\text{saw}(\text{Liz}, x) \mid x \in D_e\}

b. \[43\]_o = \text{Liz saw Bill}

c. \[43\]_cf = \begin{cases} \text{L saw B, L saw S, L saw E,} \\ \text{L saw B and S, L saw B and E, L saw S and E,} \\ \text{L saw B and S and E} \end{cases}
AL with Partial Orders

(45) Liz saw at least Bill

a. ASSERTION: \([\geq B]\)
AL with Partial Orders

(45) Liz saw at least Bill

a. ASSERTION: \([\geq B]\)

b. EPISTEMIC IMPLICATION: \(K_S[\geq B]\)
(45) Liz saw at least Bill

a. ASSERTION: \([ \geq B ]\)

b. EPISTEMIC IMPLICATION: \(K_S[ \geq B ]\)

c. \(SA([ \geq B ]) = \)

\[
\begin{cases}
[Ex \ B], \\
[Ex \ B \oplus S], [Ex \ B \oplus E], [Ex \ B \oplus S \oplus E], \\
[ \geq B \oplus S], [ \geq B \oplus E], [ \geq B \oplus S \oplus E]
\end{cases}
\]
AL with Partial Orders

(45) Liz saw at least Bill

a. **ASSERTION:** \[ \geq B \]

b. **EPISTEMIC IMPLICATION:** \[ K_S[\geq B] \]

c. **SA(\[ \geq B \]) =**
\[
\begin{align*}
&\{ [Ex \ B], \\
&[Ex \ B \oplus S], [Ex \ B \oplus E], [Ex \ B \oplus S \oplus E], \\
&[\geq B \oplus S], [\geq B \oplus E], [\geq B \oplus S \oplus E] \}
\end{align*}
\]

d. **PRIMARY IMPLICATURES:**
\[
\begin{align*}
&\neg K_S[Ex \ B] \wedge \\
&\neg K_S[Ex \ B \oplus S] \wedge \neg K_S[Ex \ B \oplus E] \wedge \neg K_S[Ex \ B \oplus S \oplus E] \wedge \\
&\neg K_S[\geq B \oplus S] \wedge \neg K_S[\geq B \oplus E] \wedge \neg K_S[\geq B \oplus S \oplus E]
\end{align*}
\]
Exhausting possibilities

The crucial difference between TOs and POs: there’s no pair of symmetric alternatives for PO associates!

\[
\begin{align*}
(46) & \quad [\geq B] \quad \Rightarrow \\
& \quad \text{exhaust the space of possibilities} \\
& \quad [\text{Ex } B] \land [\geq B \oplus S] \land [\geq B \oplus E]
\end{align*}
\]
Exhausting possibilities

The crucial difference between TOs and POs: there’s no pair of symmetric alternatives for PO associates!

Exhaust the space of possibilities

\[ (46) \quad \left[ \geq B \right] \quad \Rightarrow \quad \left[ \begin{array}{c}
\text{assertion} \\
\left[ \begin{array}{c}
\text{SA} & \left[ \begin{array}{c}
\text{SA} \\
\left[ \begin{array}{c}
\text{SA} & \left[ \begin{array}{c}
\text{SA} \\
\left[ \begin{array}{c}
\text{SA}
\end{array}\right]
\end{array}\right]
\end{array}\right]
\end{array}\right]
\end{array} \right.
\right]
\]

No entailments of the form \( \neg K_S \neg [\varphi] \) (\( \equiv P_S[\varphi] \))! E.g.: S could know that:

- it’s not the case that Liz saw only Bill.
  \( \sim S \) doesn’t know who else Liz saw
Exhausting possibilities

The crucial difference between TOs and POs: there’s no pair of symmetric alternatives for PO associates!

\[(46) \quad [\geq B] \quad \Rightarrow \quad \left[ \text{exhaust the space of possibilities} \right]
\]

\[
\begin{align*}
[\text{Ex } B] \land [\geq B \oplus S] \land [\geq B \oplus E] \\
\text{SA} \quad \text{SA} \quad \text{SA}
\end{align*}
\]

✔ No entailments of the form \(\neg K_S \neg[\varphi] \equiv P_S[\varphi]\)! E.g.: S **could** know that:

- it’s not the case that Liz saw *only* Bill.
  \(\leadsto S\) doesn’t know who else Liz saw
- it’s not the case that Liz saw *at least* Bill and Sue.
  \(\leadsto S\) doesn’t know whether Liz saw *only* Bill or Bill and Sue.
Exhausting possibilities

The crucial difference between TOs and POs: there’s no pair of symmetric alternatives for PO associates!

\[
\begin{align*}
(46) \quad & \left[ \geq B \right] \\
\Rightarrow \quad & \left[ \text{Ex } B \right] \land \left[ \geq B \oplus S \right] \land \left[ \geq B \oplus E \right] \\
\text{assertion} \quad & \text{SA} \quad \text{SA} \quad \text{SA}
\end{align*}
\]

\(\vDash\) No entailments of the form \(\neg K_S \neg [\varphi] (\equiv P_S [\varphi])\)! E.g.: S could know that:

- it’s not the case that Liz saw only Bill.
  \(\sim S\) doesn’t know who else Liz saw
- it’s not the case that Liz saw at least Bill and Sue.
  \(\sim S\) doesn’t know whether Liz saw only Bill or Bill and Sue.

\(\vDash\) However, the speaker must not know that e.g. Liz saw only Bill.
Pragmatics: Conclusion II

When AL modifies a partially ordered associate:

① No ignorance about any one particular alternative is predicted: they all may but need not constitute epistemic possibilities.
When AL modifies a partially ordered associate:

① No ignorance about any one particular alternative is predicted: they all *may but need not* constitute epistemic possibilities.

② The resulting IIs are weaker than those of AL with totally ordered associates.

✠ AL with TO: **Strong Partial Ignorance**  \(\Rightarrow\) IIs about *two* alternatives.

✠ AL with PO: **Weak Partial Ignorance**  \(\Rightarrow\) Uncertainty inferences.
Pragmatics: Conclusion II

1. **New Empirical Generalization**

The form of IIIs depends on the structural properties of AL’s associate.
Pragmatics: Conclusion II

① **New Empirical Generalization**
   The form of IIs depends on the structural properties of AL’s associate.

② **AL is only** sensitive to the structural properties of its associate.
Pragmatics: Conclusion II

① **New Empirical Generalization**
   The form of IIIs depends on the structural properties of AL’s associate.

② AL is *only* sensitive to the structural properties of its associate.

③ **New Pragmatic Calculus**
   AL involves a pragmatic reasoning procedure that relies on both focus and exhaustive alternatives:
   - EX-alternatives are required to create symmetric alternatives.
   - AL-alternatives are required to provide a measure of information strength with non-entailing scales.
Pragmatics: Conclusion II

① **New Empirical Generalization**
The form of IIs depends on the structural properties of AL’s associate.

② AL is *only* sensitive to the structural properties of its associate.

③ **New Pragmatic Calculus**
AL involves a pragmatic reasoning procedure that relies on both focus and exhaustive alternatives:

- EX-alternatives are required to create *symmetric* alternatives.
- AL-alternatives are required to provide a measure of information strength with non-entailing scales.

④ With the two-alternative system we get:

- Flexibility: we may draw IIs about any focused constituent.
- Strength of Ignorance: we account for the Partial Ignorance of AL (*viz.* Total Ignorance of disjunction).
- Non-uniformity: the status of the IIs differs across different types of associates.
CONCLUSION & RAMIFICATIONS
Pragmatics as the “science of the unsaid”.

Conclusions

What types of scales support conversational implicatures?

- Contextualism (e.g. Hirschberg 1991, 93) — “...the orderings [that support scalar implicatures] are partially [contextually] ordered sets […] and any poset can support scalar implicatures.”
- Logicism (e.g. Magri 2017, 10) — “…the algorithm for the computation of scalar implicatures must be purely logical, namely blind to common knowledge.”
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Conclusions

Contextualism
orderings

Logicism
algorithm
Conclusions: the lesson from AL

The question is ill-posed, it’s not either one or the other!
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AL, through focus association, selects an associate that induces some order, by relying on entailment, context, conventions, world knowledge...
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The question is ill-posed, it’s not either one or the other!

1. AL, through focus association, selects an associate that induces some order, by relying on entailment, context, conventions, world knowledge...

2. The pragmatic algorithm generates EX- and AL-alternatives that are more or less informative; i.e. compatible with more or less worlds; i.e. they stand in set-subset relations.
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reconciliation

We do not rely on logic to establish order, we do rely on logic to reason about order.
Looking forward: the *locus* of order

- All AL requires to be used felicitously is the **identification** of a salient order.
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<table>
<thead>
<tr>
<th>Weight</th>
<th>Sufficient Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>3kg.</td>
<td>1kg. is sufficient</td>
</tr>
<tr>
<td>2kg.</td>
<td>2kg. are sufficient</td>
</tr>
<tr>
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<td>3kg. are sufficient</td>
</tr>
</tbody>
</table>
Looking forward: the *locus* of order

(47) a. The apples weigh at least 2kg

$\sim$ S doesn’t know whether exactly 2kgs. or more.
Looking forward: the *locus* of order

(47) a. The apples weigh at least 2kg
  ~ S doesn’t know whether exactly 2kgs. or more.

b. ?At least 2kg. of apples are sufficient.
  ~ S *doesn’t know whether exactly 2kg. or less are sufficient.*
Looking forward: the *locus* of order

(47) a. The apples weigh at least 2kg
   \[ \sim S \text{ doesn’t know whether exactly 2kgs. or more.} \]
   b. At least 2kg of apples are sufficient.
      \[ \sim S \text{ doesn’t know whether exactly 2kg. or less are sufficient.} \]

(48) a. At least 20mph is fast for this road.
    b. At least 20 people are too many.

(49) a. 20mph or more is fast for this road.
    b. 20 or more people are too many.
Looking forward: the *locus* of order

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(49) a. 20mph or more is fast for this road.
    b. 20 or more people are too many.

The ordering is induced solely on the basis of its focus associate, not on the whole proposition.
Looking forward: *impossible* orderings?

Can we identify a natural class of structures that support IIIs with AL?
Looking forward: *impossible* orderings?

Can we identify a natural class of structures that support IIs with AL?

```
?pet
│     │     │     │
poodle | beagle | siamese | persian
│     │     │     │
dog    |       | cat    |
│     │     │
pet    |
```

\[ \leq \]
Looking forward: impossible orderings?

Can we identify a natural class of structures that support IIIs with AL?

(50) a. What pet did Sue get?
   b. She got at least a dog.

\[ S \text{ doesn’t know whether Sue got a poodle or a beagle } \]
Looking forward: *impossible* orderings?

```
dog ⊕ cat ⊕ bird

dog ⊕ cat
  dog
  cat

dog ⊕ bird
  dog
  bird

cat ⊕ bird
  cat
  bird
```
Looking forward: *impossible* orderings?

(51) a. What pet did Sue get?
b. She got at least a dog.

$\sim S$ doesn’t know whether Sue got a cat and a bird as well
Looking forward: *impossible* orderings?

Not any PO set would do. IIs of AL seem to prefer structures with a **unique maximal element** (i.e. a join-semilattice).
The science of the unsaid is partly the science of finding and reasoning about order.
Thank you!


(52) Every student read at least two papers.

- IIIs are derived if there is a pair of *symmetric* Stronger Alternatives.
- But ∀[Ex \(2\)] and ∀[\(\geq 3\)] are not symmetric, since ∀[\(\geq 2\)] could be true by virtue of some students reading exactly two papers while other students read more than two.
Every student read at least two papers.

- IIs are derived if there is a pair of symmetric Stronger Alternatives.
- But \( \forall [Ex \ 2] \) and \( \forall [\geq 3] \) are not symmetric, since \( \forall [\geq 2] \) could be true by virtue of some students reading exactly two papers while other students read more than two.
- The relevant interaction seems to be between \( \forall \) and \( \geq \):

\[
\begin{align*}
(52) & \quad \text{Every student read at least two papers.} \\
(53) & \quad \text{a. If you guess at least three questions, you’ll get the price.} \\
& \quad \quad \text{b. Spiders have at least two eyes.} \\
& \quad \quad \text{c. Bill \{must\}/\{has to\}/\{needs to\} read at least two papers to pass.} \\
& \quad \quad \text{d. Bring at least one of your ice-axes.}
\end{align*}
\]
(52) Every student read at least two papers.

• IIIs are derived if there is a pair of **symmetric** Stronger Alternatives.
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   b. Spiders have at least two eyes.
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• The interpretation conveying ignorance may be achieved by interpreting the universal quantifier under the scope of AL: \([\geq 2 \forall]\).
(54) At least two people came to the party.
  \[ \neg \text{It is not the case that at least three people came to the party} \]

- Prediction: IIs about two SAs, \([\text{Ex 2}]\) and \([\geq 3]\):
  \[ \neg K_S[\text{Ex 2}] \land \neg K_S[\neg \text{Ex 2}] \land \neg K_S[\geq 3] \land \neg K_S[\neg \geq 3] \]
Scalar Implicature: A problem

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  \[ \neg K_S[Ex \ 2] \land \neg K_S[Ex \ 2] \land \neg K_S[\geq 3] \land \neg K_S[\geq 3] \]

- **In principle,** any other additional alternative \( \varphi \) could be strengthened, \( K_S[\varphi] \). There are **many** such alternatives. E.g. [Ex 3] and \([\geq 4]\).

- **What if we strengthen both?** Contradiction:
  \[ K_S[Ex \ 3] \land K_S[\geq 4] \models K_S[\geq 3] \]
Scalar Implicature: Towards a solution

① Schwarz’s (2016) solution: import a consistency preservation mechanism from grammatical approaches to implicature (cf. Innocent Exclusion).

〇 It does not look very Gricean; i.e. domain-general, rational, relying on social cognition, etc.
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   - \( ES_1 \) the speaker is **maximally knowledgeable** about the question that the proposition she is uttering is making a contribution to.
     - What if the task of *at least* is precisely to preempt these assumption?
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ES₁ the speaker is **maximally knowledgeable** about the question that the proposition she is uttering is making a contribution to.

- What if the task of *at least* is precisely to preempt these assumption?

ES₂ The listener is free to consider the speaker an authority about **any** SA, unless it is in conflict with IIs.

- Then for $K_S[\geq 2]$ the listener will not infer $K_S\neg[Ex \ 3] \land K_S\neg[\geq 4]$ because this conjunction is in conflict with the $II\neg K_S[\geq 3] \land \neg K_S\neg[\geq 3]$. 
Experiment: Paradigm

- Context: Sue is teaching a class to four students: Mary, Liz, Al, and Bill. A colleague asks:
Experiment: Paradigm

- **Context**: Sue is teaching a class to four students: Mary, Liz, Al, and Bill. A colleague asks:

- **Type** Total Order (Numeral):
  - **Question**:
    How many students completed the quiz?
  - **Answer**:
    I don’t remember, at least two...
    - ...maybe one. Condition BAD
    - ...maybe more. Condition GOOD
    - ...but not only two. Condition TARGET
**Context**: Sue is teaching a class to four students: Mary, Liz, Al, and Bill. A colleague asks:

**Type** Total Order (Numeral):
- **Question**:
  How many students completed the quiz?
- **Answer**:
  I don’t remember, at least two...
  - ...maybe one.
  - ...maybe more.
  - ...but not only two.

**Type** Partial Order (Conjunction):
- **Question**:
  Who completed the quiz?
- **Answer**:
  I don’t remember, at least Mary and Liz...
  - ...maybe only Liz.
  - ...maybe somebody else too.
  - ...but not only them.
Results

Type × Condition = $p < .0001$

No effect of BAD
No effect of GOOD