Focusing on Scales[*]

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1. Introduction

It is well known that Superlative Modifiers (SMs) like *at least* and *at most* differ from their comparative counterparts (like *more than*) in that the former give rise to certain Inferences of Ignorance. These Ignorance Inferences are responsible for the illformedness of (2) compared to (1): (2a) conveys that the speaker is ignorant as to how many daughters she has, and (2b) conveys uncertainty about how many sides triangles have.[1]

(1) a. I have more than one daughters.
    b. Triangles have fewer than 11 sides.

(2) a. #I have at least two daughters.
    b. #Triangles have at most 10 sides.

Theories about how these inferences arise differ on whether they take them to follow from the semantic properties of SMs (Geurts & Nouwen 2007; Nouwen 2010) or whether they take them to be the result of a pragmatic process (Büring 2007; Coppock & Brochhagen 2013; Mayr 2013; Nouwen 2015; Schwarz 2016, a.o.).

Most works cited above have concentrated on cases where SMs modify numerals, despite the fact that they can occur in a variety of environments. This paper focuses precisely on the wide distribution of SMs. The goal is to provide a unified account of the SMs *at least* and *at most* as scalar modifiers. In particular, I follow the idea that Ignorance Inferences of SMs arise as Neo-Gricean Primary Implicatures, but I argue that computing these implicatures with SMs requires to factor in both Horn Set alternatives and focus alternatives.

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[1]I will gloss over other uses of SMs, like concessive readings of *at least*. These readings are fundamentally different from the epistemic cases in a number of respects. For instance, concessive *at least* requires that all the best alternatives be false (see Nakanishi & Rullmann 2009 and Biezma 2013 for discussion).

The account presented here formulates a classical Neo-Gricean implementation elaborating on the following ideas: (i) SMs are conventionally associated with focus (Beaver & Clark 2008, Mendia 2016), (ii) the Ignorance Inferences are equivalent to the disjuction of the corresponding equality and strict comparison (e.g., $[\geq n] \rightsquigarrow ([= n] \vee [> n])$; Büring 2007), and (iii) the calculation of implicatures with SMs requires two sets of alternatives (Mayr 2013; Schwarz 2016).

The main innovation of the calculus presented here is that the set of alternatives relevant for the Gricean calculus is provided by two independent mechanisms. In addition to the familiar substitution method within elements of a Horn Set (Horn 1972; Sauerland 2004; a.o.), a different set of alternatives is obtained by replacing the focus–bearing constituent with contextually relevant alternatives (Rooth 1992; cf. Fox & Katzir 2011). In addition, I propose that Superlative Modifiers form a Horn Set with *only*:[2]

(3) a. Horn Set for *at least*: {*at least, only*}
b. Horn Set for *at most*: {*at most, only*}

2. Ignorance is pragmatic

First, let me make a case for the pragmatic nature of the ignorance component conveyed by SMs. The usual tests of cancelability and reinforceability yield a positive result.

(4) *Cancelability*
Context: Bill has four kids. Yesterday he saw a sign at a supermarket: "Discounts for parents. To qualify you must have at least three kids." Bill reasoned as follows: I qualify: I have four kids, so I do have at least three kids.

(5) *Reinforceability*
Bill has {at least/at most} three kids, I don't know how many exactly.

More interestingly perhaps is the fact that no ignorance is conveyed when SMs appear in contexts where the Maxim of Quantity has been deactivated. This is unexpected if ignorance was not a conversational implicature. Consider (6): an SM is felicitous even though by assumption the speaker knows the exact number he is talking about (cf. Fox 2014 for disjunction).

(6) a. *Context:* In a game, my friend has to guess the number of marbles that I have hidden. I know how many I have hidden, and she knows that I have that information. I provide the following clue:
b. I have { at least / at most } five marbles.
$\rightsquigarrow$ *no ignorance about the number of marbles that I have*

[2] At the time of presenting this paper I learned that this idea has also been explored in Schwarz (2016) in the context of modified numerals.

The examples in (4) through (6) show that the ignorance conveyed by SMs is better understood as conversational implicatures that depend on the Maxim of Quantity being active.

3. Desiderata

Having established the (conversational) implicative nature of the relevant inferences, the main property of SMs that I want to highlight is their flexibility: SMs can combine with a number of scales, numeral and non–numeral. More importantly, regardless of the scale SMs associate with, Ignorance Inferences are always present, and they are always of the same form (provided that certain pragmatic assumptions hold, as explained below). The following is a sample of different kinds of scales together with an informal paraphrase of what the speaker is conveying ignorance about.[3]

(7) *Horn Set Scales*

a. At least some students came to the party.
$\rightsquigarrow$ *the speaker is ignorant about whether all students came*

b. At most a few students came to the party.
$\rightsquigarrow$ *the speaker is ignorant about whether any student came*

(8) *Cardinal Scales*

a. At least Bill came to the party.
$\rightsquigarrow$ *the speaker is ignorant about whether someone else came to the party*

b. At most Bill and Jane came to the party.
$\rightsquigarrow$ *the speaker is ignorant about whether any of them came to the party*

(9) *Lexical Scales*

a. Sue won at least the silver medal.
$\rightsquigarrow$ *the speaker is ignorant about whether Sue won the golden medal*

b. Sue won at most the silver medal.
$\rightsquigarrow$ *the speaker is ignorant about whether Sue won any medal*

(10) *Evaluative Scales* [for a preference scale ⟨ *fishbones, broccoli, candy* ⟩]

a. Bill ate at least broccoli.
$\rightsquigarrow$ *the speaker is ignorant about whether Bill ate candy*

b. Bill ate at most broccoli.
$\rightsquigarrow$ *the speaker is ignorant about whether Bill ate fishbones*

The second property of SMs that I want to draw attention to is their focus sensitivity (first discussed in Krifka 1999). SMs behave like other focusing adverbs in that they can associate with focus at a distance. When this happens, the inferences that the proposition

[3] Notice that the scales themselves may have different structures; for instance, (7) constitutes a strict order, whereas the associate of the SMs in (8) are only partially ordered. See Hirschberg (1985) for discussion.

conveys and that the addressee is allowed to infer covary with the associate of the SM (examples from Coppock & Brochhagen 2013).

(11) a. The chair { at least / at most } invited the [postdoc]$_F$ to lunch
⇝ *ignorance about whether* {*someone else/someone*} *was invited*
b. The chair { at least / at most } invited the postdoc [to lunch]$_F$
⇝ *ignorance about whether she got invited to* {*something else/anything*}

Moreover, Ignorance Inferences disappear when no focus is predicted under the scope of the SM (Westera & Brasoveanu 2014). For instance in the short dialog in (12), the sentence (12b) directly answers the question in (12a): Focus in (12b) *as an answer* to (12a) is predicted to be on [ten students] (see e.g., Rooth 1992), which in turn correlates with what the speaker is ignorant about.

(12) a. Exactly how many students took Linguistics 101?
b. {At least/At most } ten students took Linguistics 101. [=(13b)]
⇝ *ignorance about the exact number*

In contrast, in (13b) the associate of the SM does not directly answer (13a) and no constituent is predicted to bear focus. Correspondingly, the listener need not derive an Ignorance Inference.

(13) a. Did {at least/at most } ten students take Linguistics 101?
b. {At least/At most } ten students took Linguistics 101. [=(12b)]
⇝*no ignorance about the exact number*

In sum, SMs can associate with focus at a distance and Ignorance Inferences covary with the constituent that bears focus; in the absence of one such constituent, no Ignorance Inferences arise. The rest of the paper is devoted to provide an account of Ignorance Inferences as conversational implicatures that is flexible enough to accommodate the variety of environments in which SMs may appear.

4. Background

Assume that K and P stand for the epistemic certainty and possibility operators (Hintikka 1962). Given the properties that Hintikka (1962) ascribed to K and P, $\neg K_S \neg[\phi]$ is equivalent to $P[\phi]$, and constitutes an epistemic possibility compatible with all the speaker knows. Assume further that by the principle of Epistemic Implication of Hintikka (1962), utterance of a sentence ϕ by a speaker S commits S to the knowledge of ϕ (expressed as $K_S\phi$). Consequently, the addressee is allowed to infer that the utterance of ϕ by S implicates that $K_S\phi$. This is, of course, provided that we are dealing with a cooperative speaker, one that, minimally, observes the Maxims of Quality or some version thereof (Grice 1989).

(14) MAXIMS OF QUALITY

a. Do not say what you believe to be false.
b. Do not say what you do not have evidence for.

Now we can be more explicit about what it means to be ignorant. To be ignorant about a proposition ϕ requires (i) to not know that ϕ and (ii) to not know that $\neg\phi$. This corresponds to the "signature of ignorance", as defined below.

(15) SIGNATURE OF IGNORANCE
$\neg K_S \phi \wedge \neg K_S \neg\phi$ *equivalent to* $P_S \phi \wedge P_S \neg\phi$

Now the goal is to derive Ignorance Inferences for SMs that conform to the template in (15). The inferences will arise as a converstional implicatures that are derived reasoning under the assumption that we are dealing with a cooperative speaker, and so under the assumption that some version of the Maxim of Quantity is at work. In this case, Quantity is defined in terms of asymmetric entailment.

(16) MAXIM OF QUANTITY
If two propositions ϕ and ψ are such that (i) ϕ and ψ are relevant, (ii) $\phi \rightarrow \psi \wedge \neg(\psi \rightarrow \phi)$, and (iii) $K_S \phi$ and $K_S \psi$, the speaker should choose ϕ over ψ.

It is useful to define the notion of Stronger Alternative: an alternative proposition ψ to the proposition ϕ expressed by the utterance which asymmetrically entails ϕ. We can now derive a Primary Implicature, a negative inference that the speaker does not know the truth of some stronger alternative proposition to the one she expressed by her utterance. Finally, I refer to the Implicature Base as the conjunction of the Epistemic Implication and the Primary Implicatures.

(17) a. PRIMARY IMPLICATURE:
The inference that $\neg K_S \psi$, for some Stronger Alternative ψ of ϕ.
b. IMPLICATURE BASE:
The Epistemic Implication conjoined with its Primary Implicatures.

5. Analysis

The previous section has set what the formal principles are that are responsible for an Ignorance Inference. In what follows, I show how considering both Horn Set alternatives and focus alternatives provides the right kind of input to the implicature calculating algorithm so that it derives Ignorance Inferences of the right form.

5.1 Step 1: Focus

I take it that SMs are scalar modifiers interpreted relative to some focalized constituent.[4] For simplicity, I assume a propositional version of SMs, such that they can directly take sets of propositions as arguments (Büring 2007).

(18) a. $[\![\textit{at least}]\!] = \lambda C_{\langle st,t\rangle}.\lambda p_{\langle st\rangle}.\lambda w : \exists q[q \in C \wedge p \leq q \wedge q(w)]$
there is a proposition q *in* C *which is at least as strong as the prejacent* p *such that* q *is true in* w

b. $[\![\textit{at most}]\!] = \lambda C_{\langle st,t\rangle}.\lambda p_{\langle st\rangle}.\lambda w : \forall q[q \in C \wedge q(w) \rightarrow q \leq p]$
for every proposition q *in* C*, if* q *is true, then* q *is at most as strong as the prejacent* p *in* w

Following Rooth's analysis of focus, the focus operator "∼" presupposes that there is some contextually relevant set of alternatives C which is a subset of the focus value $[\![\mathrm{XP}]\!]^f$ containing, minimally, $[\![\mathrm{XP}]\!]^o$ and one other element.

(19) a. Sue won {at least / at most } [a silver medal]$_F$

b. LF: $[_{S^1}$ SM C $[_{S^2}$ $[_{S^3}$ Sue won [a silver medal]$_F$] $\sim C$]]

(20) a. $[\![$ Sue won [a silver medal]$_F]\!]^f = \{\textit{Sue won } x \mid x \in D_e\}$,
where $D_e = \{\textit{a bronze medal, a silver medal, a gold medal}\}$

b. $$[\![(19\text{a})]\!]^f = \left\{\begin{array}{l} \textit{Sue won a bronze medal,} \\ \textit{Sue won a silver medal,} \\ \textit{Sue won a gold medal} \end{array}\right\}$$

c. $[\![(19\text{a})]\!]^o =$ *Sue won a silver medal*

(21) *Asserted content of propositions with SMs*

a. $[\![(19\text{a})_{\textit{at least}}]\!] = \lambda w : \exists q[q \in [\![(19\text{a})]\!]^f \wedge [\![(19\text{a})]\!]^o \leq q \wedge q(w)]$

b. $[\![(19\text{a})_{\textit{at most}}]\!] = \lambda w : \forall q[q \in [\![(19\text{a})]\!]^f \wedge q(w) \rightarrow q \leq [\![(19\text{a})]\!]^o]$

5.2 Step 2: Generating alternatives

In order to calculate implicatures in a Neo-Gricean framework, alternative propositions have to be ordered. This can happen in various ways. For instance, an ordering can be provided by asymmetric entailment relations among propositional focus alternatives. The semantics of focus delivers an ordinary semantic value and a focus semantic value that consists of a set of alternative propositions. Then we can use this set of propositions to reason about plausible and more informative alternative propositions that the speaker could have uttered –just like we usually do in routine Neo-Gricean pragmatics. I suggest that this constitutes the first set of relevant alternative propositions that is factored into the pragmatic

[4]The reader is referred to Mendia (2016) for a number of arguments that SMs' association with focus is conventional and that their behavior is in this sense very similar to other focusing adverbs like *only* or *even*.

calculus. For instance, for a sentence like *Sue won at least a silver medal*, the focus value provides the set of propositional alternatives in (22), and from this set we can identify the most informative alternatives, as in (23).[5]

(22) *Alternatives from focus*
$\texttt{Alt}_{\texttt{foc}}$(*Sue won at least a silver medal*) = $\{[\![\text{at least } p]\!] \mid p \in [\![(19a)]\!]^f\}$
$$\left\{\begin{array}{l}\textit{Sue won at least a bronze medal},\\ \textit{Sue won at least a silver medal},\\ \textit{Sue won at least a gold medal}\end{array}\right\}$$

(23) *Stronger Alternatives from focus*
$\texttt{StrAlt}_{\texttt{foc}}$(*Sue won at least a silver medal*) = {*Sue won at least a gold medal*}

Focus alternatives are not the only way alternative propositions can be ordered. In the case of Horn Sets, alternatives are ordered by virtue of the lexical properties of its scale-mates. This is no different in the case of SMs. As advanced in the introduction, I suggest that each of the SMs participate in a Horn Set together with *only*, as in (3) above. Traditional Horn Sets like {*some, all*} or {*or, and*} are generalizations over lexical items that stand in a relation of asymmetric entailment. Since SMs stand in an asymmetric entailment relation with *only*, this seems a plausible option. So, following the usual substitution method (e.g., Sauerland 2004) scalar alternatives are generated from the set of focus alternatives by swapping SMs with *only*.[6] In this case, we generate the new set of alternative propositions in (24) by trading the SM for *only*, and then we pick those alternatives that asymmetrically entail the prejacent to generate a second set of Stronger Alternatives, as in (25).

(24) *Alternatives from Horn Sets*
$\texttt{Alt}_{\texttt{hs}}$(*Sue won at least a silver medal*) = $\{[\![\text{only } p]\!] \mid p \in [\![(19a)]\!]^f\}$
$$= \left\{\begin{array}{l}\textit{Sue won only a bronze medal},\\ \textit{Sue won only a silver medal},\\ \textit{Sue won only a gold medal}\end{array}\right\}$$

(25) *Stronger Alternatives from Horn Sets*
$\texttt{StrAlt}_{\texttt{hs}}$(*Sue won at least a silver medal*) = $\left\{\begin{array}{l}\textit{Sue won only a silver medal},\\ \textit{Sue won only a gold medal}\end{array}\right\}$

[5] Due to the limited space, I will only illustrate the derivation with the SM *at least*, but everything I show carries over to *at most* too, as the reader is invited to check.

[6] One may wonder whether the presuppositional properties of *only* first discussed in the classic analysis of Horn (1969) could interfere with the implicature calculation mechanism. While I do not have space to address this worry here, other formulations of this idea are also possible. For instance, as Schwarz (2016) mentions in a footnote, by replacing *only* with the silent exhaustivity operator *Exh* (Fox (2007), a.o.). For now, assume with Rooth (1992) that $[\![\text{only}]\!] = \lambda C_{\langle st,t\rangle}.\lambda p_{\langle st\rangle}.\lambda w : \forall q[q \in C \wedge q(w) \leftrightarrow p = q]$.

Thus, Horn Sets provide the second relevant set of alternatives that feeds the implicature calculation mechanism. Putting together both sets of Stronger Alternatives in (23) and (25), we get the final set of Stronger Alternatives over which we calculate implicatures.

(26) *Stronger Alternatives* – FINAL

$$\texttt{StrAlt}_{\texttt{foc+hs}}(\textit{Sue won at least a silver medal}) = \left\{\begin{array}{l}\textit{Sue won at least a gold medal},\\ \textit{Sue won only a silver medal},\\ \textit{Sue won only a gold medal}\end{array}\right\}$$

5.3 Step 3: Calculating implicatures

The set of Stronger Alternatives calculated above is sufficient to derive the right kind of Ignorance Inferences simply by following a Neo-Gricean style reasoning about conversational cooperation. Take for instance the following assertion and the corresponding inference that the speaker is knowledgeable about the proposition in question:[7]

(27) a. *Sue won at least a bronze medal* [$\geq$ `Bronze`]
b. K_S(*Sue won at least a bronze medal*) K_S[$\geq$ `Bronze`]

The set of Stronger Alternatives is derived like explained in the previous two subsections, by calculating Stronger Alternatives from two different sources: focus alternatives and Horn Set alternatives.

(28) *Stronger Alternatives from Horn Sets*:

$$\texttt{StrAlt}_{\texttt{hs}}(\textit{Sue won at least a bronze medal}) = \left\{\begin{array}{l}\textit{Sue won only a gold medal},\\ \textit{Sue won only a silver medal},\\ \textit{Sue won only a bronze medal}\end{array}\right\} \quad \begin{array}{r}[O\ \texttt{Gold}]\\ [O\ \texttt{Silver}]\\ [O\ \texttt{Bronze}]\end{array}$$

(29) *Stronger Alternatives from focus*:

$$\texttt{StrAlt}_{\texttt{foc}}(\textit{Sue won at least a bronze medal}) = \left\{\begin{array}{l}\textit{Sue won at least a gold medal},\\ \textit{Sue won at least a silver medal}\end{array}\right\} \quad \begin{array}{r}[\geq \texttt{Gold}]\\ [\geq \texttt{Silver}]\end{array}$$

Following the standard Neo-Gricean practice, we derive Primary Implicatures of the proposition in (27) by negating propositions in the set of Stronger Alternatives.

[7] I will make use of the following notational conventions. I enclose propositions in square brackets, such that [ϕ] stands for some proposition containing an expression ϕ. For instance, [4] stands for the proposition *4 students came*, and [`Al`$\oplus$`Mary`], or [`A`$\oplus$`M`] stand for *Al and Mary came*, and so on. For SMs I use [$\geq$ ϕ] for [*at least* ϕ], [$\leq$ ϕ] for [*at most* ϕ] and [O ϕ] for [*only/exactly* ϕ].

(30) *Primary Implicatures*:

$$\left\{\begin{array}{l} \neg K_S(\textit{Sue won at least a gold medal}), \\ \neg K_S(\textit{Sue won at least a silver medal}), \\ \neg K_S(\textit{Sue won only a gold medal}), \\ \neg K_S(\textit{Sue won only a silver medal}), \\ \neg K_S(\textit{Sue won only a bronze medal}) \end{array}\right\} \qquad \begin{array}{r} \neg K_S[\geq \texttt{Gold}] \\ \neg K_S[\geq \texttt{Silver}] \\ \neg K_S[O\ \texttt{Gold}] \\ \neg K_S[O\ \texttt{Silver}] \\ \neg K_S[O\ \texttt{Bronze}] \end{array}$$

Finally, the last step is to consider the Primary Implicature in conjunction with the assertion itself. This constitutes the Implicature Base of (27).

(31) *Implicature Base:* $K_S[\geq \texttt{Bronze}] \wedge$ (27b) $\wedge$ (30)
$\neg K_S[\geq \texttt{Gold}] \wedge \neg K_S[\geq \texttt{Silver}] \wedge$
$\neg K_S[O\ \texttt{Gold}] \wedge \neg K_S[O\ \texttt{Silver}] \wedge \neg K_S[O\ \texttt{Bronze}]$

The calculation is over: the Implicature Base calculated above entails that two –and only two– of the Stronger Alternatives are epistemic possibilities for the speaker:

(32) *Entailments of the Implicature Base*:

a. $\neg K_S \neg[O\ \texttt{Bronze}]$ $\quad\leftrightarrow P_S[O\ \texttt{Bronze}]$

b. $\neg K_S \neg[\geq \texttt{Silver}]$ $\quad\leftrightarrow P_S[\geq \texttt{Silver}]$

To see why this is the case, consider first (32a). If $\neg K_S \neg[O\ \texttt{Bronze}]$ were not true, as in $K_S \neg[O\ \texttt{Bronze}]$, it would entail that $K_S[\geq \texttt{Silver}]$ is true, given our assumption that $K_S[\geq \texttt{Bronze}]$ holds. But $K_S[\geq \texttt{Silver}]$ directly contradicts the Primary Implicature that $\neg K_S[\geq \texttt{Silver}]$, rendering the Implicature Base inconsistent. Thus, it must be the case that the proposition $\neg K_S \neg[O\ \texttt{Bronze}]$ is true. A similar reasoning shows that the second entailment $\neg K_S \neg[\geq \texttt{Silver}]$ also goes through. If $\neg K_S \neg[\geq \texttt{Silver}]$ were not true, $K_S \neg[\geq \texttt{Silver}]$, it would entail that $K_S[O\ \texttt{Bronze}]$ is true, which contradicts the Primary Implicature that $\neg K_S[O\ \texttt{Bronze}]$.

No other epistemic possibilities are entailed. For instance, take the alternative proposition that *Sue won only a silver medal*, $[O\ \texttt{Silver}]$. The hearer will deduce a Primary Implicature of the form $\neg K_S[O\ \texttt{Silver}]$. In this case, the corresponding epistemic possibility, $\neg K_S \neg[O\ \texttt{Silver}]$, is a contingent statement, not entailed by the Implicature Base. In fact, one could negate it, $K_S \neg[O\ \texttt{Silver}]$, without fear of contradicting any Primary Implicature or entailing any other relevant Stronger Alternative proposition.

The final step is simply to acknowledge that the Primary Implicatures together with the entailments of the Implicature Base provide us with a set of inferences that are formally identical to the Signature of Ignorance we defined in (15) above.

(33) a. Primary Implicature + $\quad\neg K_S[O\ \texttt{Bronze}]$

b. Entailment of Implicature Base = $\quad\neg K_S \neg[O\ \texttt{Bronze}]$

c. Ignorance Implicature: $\boxed{\neg K_S[O\ \texttt{Bronze}]} \wedge \boxed{\neg K_S \neg[O\ \texttt{Bronze}]}$

d. Signature of Ignorance: $\boxed{\neg K_S[\phi]} \wedge \boxed{\neg K_S \neg[\phi]}$

The same holds *mutatis mutandis* for the second epistemic possibility that is entailed by the Implicature Base, $\neg K_S \neg[\geq \texttt{Silver}]$. Together with the Primary Implicatures that $\neg K_S[O\ \texttt{Bronze}]$ and $\neg K_S[\geq \texttt{Silver}]$ both entailments consitute Ignorance Implicatures as defined in (15). Therefore, the propositions $[O\ \texttt{Bronze}]$ and $[\geq \texttt{Silver}]$ must be epistemic possibilities compatible with all the speaker knows.

6. Discussion

These results seem to be correct. By factoring in focus alternatives in the calculation of implicatures, two facts about SMs and their Ignorance Inferences are captured: that these inferences arise with a wide variety of scales and that they are always about the focus bearing constituent in the sentence. Moreover, these inferences are derived as conversational implicatures that depend on the assumption that all the pragmatic principles are in place. This is, again, a good result, since Ignorance Inferences are cancellable, reinforceable and they rely on the assumption that the speaker is being maximally informative: dropping the Maxim of Quantity makes them disappear, while a requirement to be maximally informative, either by discursive pressures or by background assumptions, triggers them. Finally, the calculation proceeds all the same for both SMs, *at least* and *at most*.

6.1 Partial Ignorance

An advantage of the present account is that the predicted inferences convey Partial Ignorance, as opposed to Total Ignorance (Nouwen 2015). The difference lies in whether Ignorance is derived for all the Stronger Alternatives or for just a subset of them. In the case of disjunction, for instance, Ignorance Inferences are Total, since all the individual disjuncts constitute epistemic possibilities for the speaker (Alonso-Ovalle 2006). This is why a discourse like the (34) below is odd.

(34) a. Who brought the cheese?
b. #Mary, Al, or Sue, but I'm sure that it wasn't Al.

In the case of SMs, however, this is not so. If ignorance was Total, a proposition like *Al brought at least two cakes* would convey Ignorance about every number above two. But this is not the case, as illustrated by the following examples.

(35) a. *Context:* I know that John is either an assistant professor or a full professor. I say: "John is at least an assistant professor".
b. Partial Ignorance: *the speaker is ignorant about whether John is an associate professor or some higher rank* [from Büring 2007]
c. Total Ignorance: *the speaker is ignorant about whether John is an assistant, associate, or full professor* [from Coppock & Brochhagen 2013, p.10][8]

(36) a. *Context:* I know that 3, 5 or 7 boys left. I say: "At least three boys left".
b. Partial Ignorance: *the speaker is ignorant about whether exactly three boys or more than three boys left.* [from Büring 2007]
c. Total Ignorance: *for any* n *larger than three, the speaker is ignorant about whether* n*–many boys left* [from Mayr 2013, p.158][8]

In the context of (35a) ignorance about John's rank cannot be Total, as suggested by (35c), since by assumption the speaker is not ignorant as to whether John is an associate professor (he knows he is not). Similarly, should not be predicted to convey that the speaker is ignorant as to whether four boys left, as provided by (36c).

6.2 Ignorance is not uniform

These results make, in addition, two sets of different predictions depending on how the associate of the SM is structured. In the working example presented in section §5, the associate of *at least* is Totally Ordered; i.e., every element in the pragmatic scale {*bronze medal*, *silver medal*, *gold medal*} either asymmetrically entails or is asymmetrically entailed by every other element. But this is not so in the case of plural (conjunctive) associates in Cardinal scales (see (8) above). In these cases, the associate of the SM constitutes a join-sublattice (Link 1983), and so it corresponds to a Partial Order, instead of a Total Order. As a consequence, not every element in the denotation of conjunctive plurals will stand in an asymmetric entailment relation with every other element.

To see why this consequence is important, consider first the case of Totally Ordered associates of SMs, like the pragmatic scale discussed in §5 or the more common numeral scale. Two main predictions are made about the kind of knowledge that is expected from a speaker uttering a sentence containing an SM associated with a Totally Ordered complement. We expect the following to *mandatorily* constitute epistemic possibilities: (i) the exhaustive interpretation of the proposition expressed by the prejacent, and (ii) the immediately Stronger Alternative proposition to the prejacent. That is, when uttering a sentence with an SM there must be two possibilities that the speaker has to consider to be true. These constitute the *assertibility conditions* of SMs with Totally Ordered associates:

(37) A proposition ϕ containing *at least* is assertible by a speaker S if

a. $\neg K_S \neg[O\,\phi]$
(The exhaustive interpretation of the prejacent is compatible with all S knows.)

b. $\neg K_S \neg[>\phi]$
(The immediately stronger alternative is compatible with all S knows.)

c. $K_S \neg[<\phi]$
(It is not the case that there is a true weaker alternative.)

[8] To be sure: the point here is not that Mayr (2013) and Coppock & Brochhagen (2013) argue for Total Ignorance (nor that they derive it); however, they do paraphrase the implicatures as shown, which is illustrative of the fact there is not much clarity in the literature about what the exact form of these implicatures should be.

These predictions seem to be right: (38a) and (38b) would not be expected from a collaborative speaker because they violate these requirements. No other mandatory epistemic possibilities are predicted, and so the wellformedness of sentences like (38c) is also accounted for.

(38) Bill ate at least two apples, but he didn't eat...

a. #{exactly / only / just} two.
b. #{more than two / at least three}
c. {exactly three / exactly four / three or four / between three and six...}

The situation is different with Partially Ordered associates. For instance, take the sentence *at least Bill came to the party*, uttered in a context where the salient individuals are *Bill, Sue* and *Liz*. In such case, *at least* associates with *Bill*, which is an element of the salient plurality of individuals (say {*Bill, Sue, Liz*}) that the focus value of provides. This plurality, which takes the form of a join-sublattice, contains two pluralities that (i) are not ordered with respect to each other, and that (ii) are minimally more informative than *Bill*: $Bill \oplus Sue$ and $Bill \oplus Liz$. As a consequence, the relevant Implicature Base that is calculated contains not two, but three Primary Implicatures that, conjoined, exhaust the space of possibilities expressed by the prejacent.

(39) Implicature Base: $K_S[\geq \mathtt{B}] \;\wedge\; \neg K_S[O\,\mathtt{B}] \wedge \neg K_S[\geq \mathtt{B} \oplus \mathtt{S}] \wedge \neg K_S[\geq \mathtt{B} \oplus \mathtt{L}]$

Now we could negate any one of the Primary Implicatures without contradicting nor entailing the truth any other. Thus, suppose that the speaker knew that it is not the case only Bill came to the party, $K_S \neg[O\,\mathtt{B}]$. As the reader can check, conjoining this assumption with the Implicature Base results in a contingent set of propositions: all it says is that the speaker knows that somebody else besides Bill came to the party, but she does not know who exactly:

(40) $K_S[\geq \mathtt{B}] \;\bigwedge\; K_S \neg[O\,\mathtt{B}] \;\bigwedge \neg K_S[O\,\mathtt{B}] \wedge \neg K_S[\geq \mathtt{B} \oplus \mathtt{S}] \wedge \neg K_S[\geq \mathtt{B} \oplus \mathtt{L}]$

As a consequence, this account predicts that assertibility conditions of utterances containing SMs vary depending on how the associate is structured.

7. Conclusion

The account of the Ignorance Inferences that come with SMs defended in this paper is based on (i) a basic epistemic logic and the assumptions that (ii) SMs are conventionally associated with focus and that (iii) we need to factor in alternatives generated from two different mechanisms, focus alternatives and Horn Sets. In doing so, it is argued the analysis defended here improves on previous approaches in a number of respects. It replicates the results obtained by Büring (2007) by deriving Ignorance Inferences in a principled way, without appealing to a syncategorematic disjunctive definitions of SMs. Thus, the resemblance of Ignorance Inferences with SMs and simple disjunctions is independently motivated. Since there is not need to assume a special status for SMs, assumptions like SMs being "inquisitive" (Coppock & Brochhagen 2013) may also follow from independent principles. In addition, the analysis improves on Schwarz (2016) in that it is extended to (i) *at most*, to (ii) a variety of scales other than the numeral, and (iii) it makes explicit the connection between SMs and focus.

A somewhat surprising consequence of the view defended in this paper is that Ignorance Inferences are predicted to vary depending on the structural properties of the SMs' associate. If the associate of the SM is Totally Ordered, it is predicted that the exhaustive interpretation of the prejacent must mandatorily constitute an epistemic possibility for the speaker. In that case, the present analysis correctly predicts what the right assertibility conditions of sentences with SMs are. However, if the associate of SMs is Partially Ordered, the exhaustive interpretation of the prejacent can but need not constitute an epistemic possibility for the speaker. I leave an in depth exploration of these predictions for a future occasion. In both cases, the account derives Ignorance Inferences of the correct kind (Partial Ignorance as opposed to Total Ignorance).

The paper has implications for the semantic properties of SMs as well. There is an ongoing debate in the literature as to whether superlatives are to be treated as degree constructions (e.g, Hackl 2000, Nouwen 2010) or focus–sensitive operators (e.g., Krifka 1999; Beck 2012). In looking at Ignorance Inferences, this paper makes a case for the focus–sensitive approach to SMs. I leave open the question as to whether it is plausible to seek a unifying approach.

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